Global Positioning System (GPS) Fundamentals

February 3, 1999

Outline

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  - **New signals**
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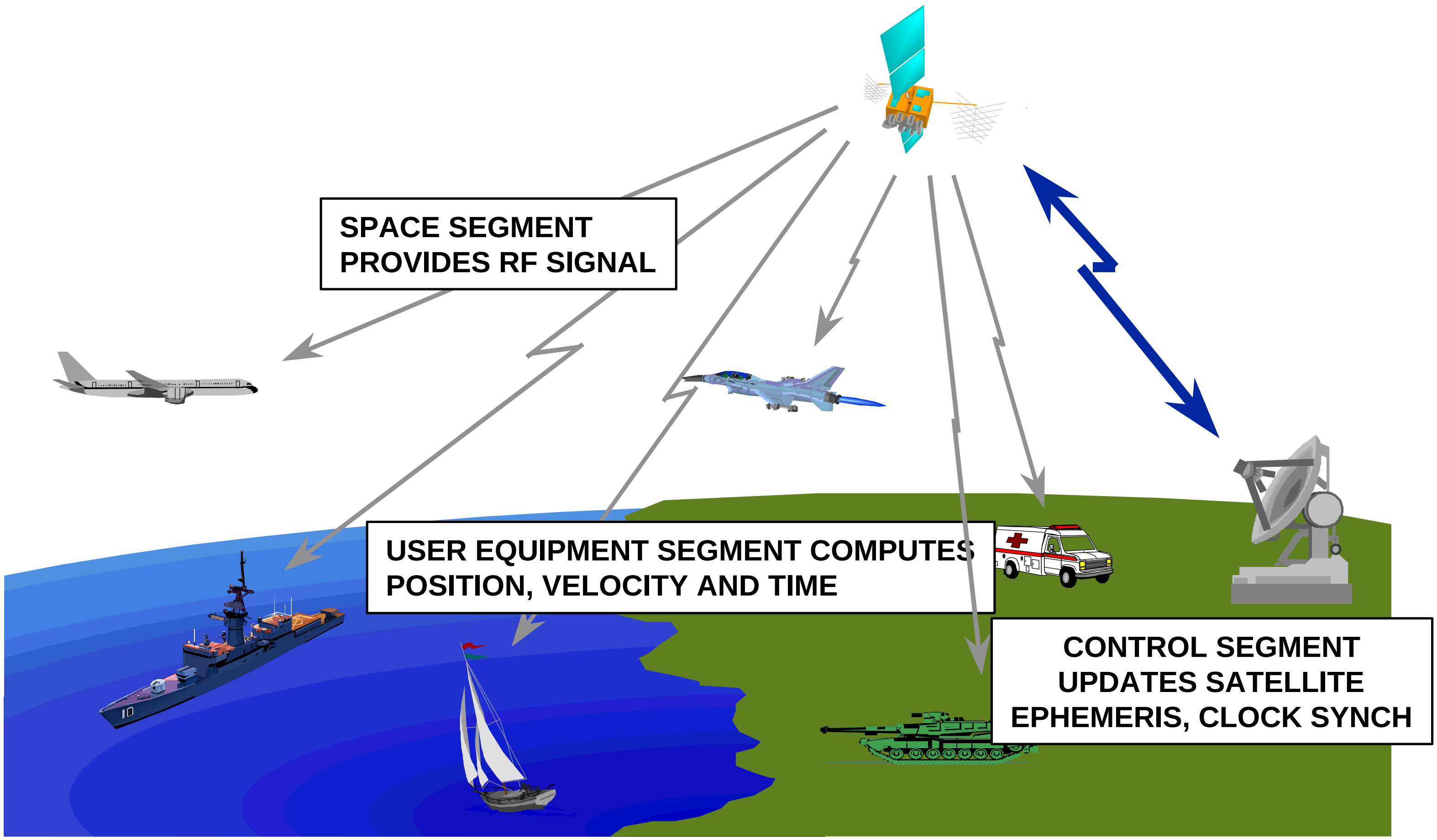
- **Cost considerations**

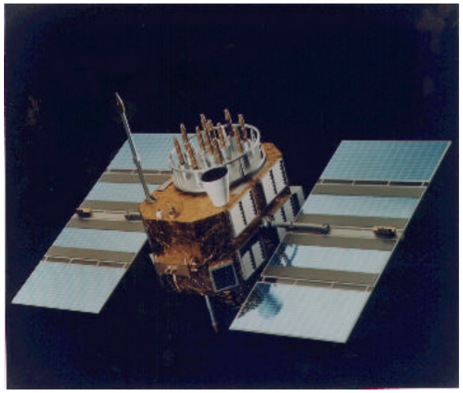
GPS Components

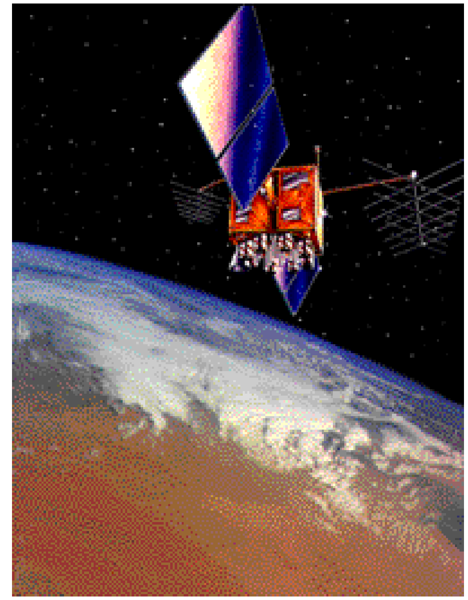


GPS Satellite Constellation

- 24 satellites
- 6-11 satellites always in view
- 11 hr, 58 min orbit (semi-synchronous)
- 10,900 nm altitude
- 6 orbit planes, 4 satellites per plane
- 55 degree inclination

GPS Satellites





Block IIR (Production)

- Lockheed Martin
- 21 satellite procurement
- First launch: 17 Jan 97
- Successful launch: 22 Jul 97

Block II/IIA (Operational)

- Boeing North American
- 28 satellite procurement
- First launch: 14 Feb 89
- Last launch: 5 Nov 97

Block IIF (Under Development)

- Boeing North American
- 30 satellite procurement
    - Procured 6 satellites with first option
    - Further options for 3 satellites each year with next option in Dec 99
- Includes opportunity for growth from baseline

GPS User Equipment

1. Manpack (RPU-1)
2. Portable Lightweight GPS Receiver (PLGR)
3. Receiver 3A
4. Receiver 3S
5. Standard Control Display Unit (CDU)
6. Fixed Reception Pattern Antenna (FRPA)
7. 1553 Receiver OH

7 A. ARINC 429 Receiver UH
8. FRPA Ground Plane (FRPAGP)
9. Control Reception Pattern Antenna (CRPA)
10. Miniature Airborne GPS Receiver (MAGR)
11. Antenna Electronics AE-1
12. Antenna Electronics AE-4

Positioning Services

- Standard Positioning Service (SPS)
  - C/A-code (coarse acquisition)
  - L1 (1575 MHz) frequency band
  - Civil use for positioning; military use for code acquisition
  - Selective Availability (SA) degraded accuracy
  - 1 msec chipping rate (1.023 chips/sec)

- Precise Positioning Service (PPS)
  - P-code (precise)
  - L1 and L2 (1227 MHz) bands
  - Military use (called Y-code when encrypted)
  - Civil access through codeless tracking techniques
  - Military access to data to compensate for the effects of SA
  - 0.1 msec chipping rate (10.23 chips/sec)

Applications

Military

Smart Weapon Guidance
Air, Land and Sea Navigation
Air Traffic Control
Precision Approach

Civilian

Car Navigation
Air, Land and Sea Navigation
Air Traffic Control
Precision Approach
Bus/Truck Fleet Monitoring
Surveying
Clock Calibration
Emergency Response
Agriculture
Communications

Waveforms

$$L1(t) = \alpha_1 P(t)D(t)\cos(2\pi f_1 t) + \alpha_1 C/A(t)D(t)\sin(2\pi f_1 t)$$
$$L2(t) = \alpha_2 P(t)D(t)\cos(2\pi f_2 t)$$

P(t) = P-code
D(t) = navigational (data) message
C/A(t) = C/A-code

- **C/A- and P-codes are:**
  - **unique to each satellite**
  - **C/A uses a "Gold" code pseudo random noise (PRN) sequence to minimize mutual correlation**
  - **Binary Phase Shift Key (BPSK) modulated on L1 and L2 carriers**

Pseudorange Measurement

$R_i = c(t_r - t_s)$; range from receiver to satellite i

- **c = speed of light ($3*10^8$ m/s)**
- **t_s = satellite transmission time**
  - **- coarse estimate is provided by the hand-over-word (HOW) encoded in each data message subframe and gives the transmission time of the next subframe (subframes are transmitted once every 6 secs)**
  - **- estimate is refined by using a second-order polynomial model of the satellite clock drift with parameters encoded in the first of the five subframes in the data message (data messages are transmitted once every 30 secs)**
- **t_r = time of receipt of code by receiver**

Position Determination

$$R_i = c(t_r - t_s) = [\ (x_r - x_i)^2 + (y_r - y_i)^2 + (z_r - z_i)^2\ ]^{1/2} + c(\delta_{rc} + \delta_{sc})$$

(x_i, y_i, z_i) = satellite position (known from ephemeris data)
(x_r, y_r, z_r) = receiver position (unknown)
δ_{rc} = receiver clock bias (unknown)
δ_{sc} = satellite clock bias (known from drift model)

Since we have four unknowns we need at least four equations to solve for the receiver position, i.e., four different satellite pseudorange measurements R_i, i=1,..,4

Position Determination (cont.)

Unfortunately equations are nonlinear -- need to linearize

$$\begin{aligned} R_i &= [\,(x_r - x_i)^2 + (y_r - y_i)^2 + (z_i - z_r)^2\,]^{1/2} + c(\delta_{rc} + \delta_{ic}) \\ &= f_i(x_r, y_r, z_r) + c(\delta_{rc} + \delta_{sc}) \\ &= f_i(x_{r0} + \Delta x_r, y_{r0} + \Delta y_r, z_{r0} + \Delta z_r) + c(\delta_{rc} + \delta_{ic}) \end{aligned}$$

using a first order Taylor series approximation about the receiver position guess (x_{r0}, y_{r0}, z_{r0}) gives

$$\begin{aligned} R_i = f_i(x_{r0}, y_{r0}, z_{r0}) &+ (d\, f_i(x_{r0}, y_{r0}, z_{r0})\, /dx_{r0})\, \Delta x_r + \\ &+ (d\, f_i(x_{r0}, y_{r0}, z_{r0})\, /dy_{r0})\, \Delta y_r + \\ &+ (d\, f_i(x_{r0}, y_{r0}, z_{r0})\, /dz_{r0})\, \Delta z_r + c(\delta_{rc} + \delta_{ic}) \end{aligned}$$

Position Determination (cont.)

the partials look like ...

$$d\, f_i(x_{r0}, y_{r0}, z_{r0}) / dx_{r0} = (x_{r0}-x_{si})/f_i(x_{r0}, y_{r0}, z_{r0})$$

so,

$$\begin{aligned} b_i = R_i - f_i(x_{r0}, y_{r0}, z_{r0}) - c\delta_{ic} = &\\ &[(x_{r0}-x_i)/f_i(x_{r0}, y_{r0}, z_{r0})]\Delta x_r + \\ &[(y_{r0}-y_i)/f_i(x_{r0}, y_{r0}, z_{r0})]\Delta y_r + \\ &[(z_{r0}-z_i)/f_i(x_{r0}, y_{r0}, z_{r0})]\Delta z_r + c\delta_{rc} \end{aligned}$$

$$b_i = a_{i1}\, p_1 + a_{i2}\, p_2 + a_{i3}\, p_3 + a_{i4}\, p_4 = \underline{a}_i^T \underline{p}$$

Position Determination (cont.)

In matrix form,

$$\underline{b} = A\,\underline{p}$$

if N (number of satellites in view) is 4 then the solution is given by

$$\underline{p} = A^{-1}\underline{b}$$

if N>4 then it is an overdetermined system and a least squares solution is used

$$\boxed{\underline{p} = (A^T A)^{-1} A^T \underline{b}.}$$

Velocity Determination

$$f_R = f_T\,[\,1 - (\underline{v}_r \bullet \underline{a})/c\,] = f_T + \Delta f; \qquad \underline{v}_r = \underline{v} - \underline{\dot{u}}$$

$$\Delta f = -f_T\,[\,(\underline{v} - \underline{\dot{u}}) \bullet \underline{a}\,]/c$$

$\underline{v}$ = satellite velocity vector (known from ephemeris data)
$\underline{u}$ = receiver position (known from position determination calculation)
$\underline{\dot{u}}$ = receiver veocity vector (unknown)
$\underline{a}$ = unit vector pointing from the satellite to the receiver (known)
c = speed of light (constant)
f_R = carrier frequency at receiver
f_T = transmitted carrier frequency
Δf = Doppler shift

Velocity Determination (cont.)

For satellite j;

$$f_{Tj} = f_{L1} + \Delta f_{Tj}$$

where Δf_{Tj} is determined from navigation message. Also,

$$f_{Rj} = f_j + f_j \dot{t}_u$$

where f_j is the receiver measured frequency and $\dot{t}_u$ is the user's clock drift rate. So,

$$f_{Rj} = f_j (1 + \dot{t}_u) = f_{Tj} [\, 1 - (\underline{v}_r \bullet \underline{a})/c \,]$$

Velocity Determination (cont.)

Consolidating terms and expanding the dot product gives

$$d_j = a_{jx}\,\dot{u}_x + a_{jy}\,\dot{u}_y + a_{jz}\,\dot{u}_z - (c\, f_j\, \dot{t}_u / f_{Tj})$$

where,

$$d_j = c(f_j - f_{Tj})/f_{Tj} + v_{jx}\, a_{jx} + v_{jy}\, a_{jy} + v_{jz}\, a_{jz}$$

in matrix form

$$\underline{d} = A\underline{\dot{u}}$$

$$\text{so} \qquad \underline{\dot{u}} = A^{-1}\underline{d}$$

Dilution of Precision (DOP)

By standard regression analysis -

$$\text{Var.}(p) = (A^T A)^{-1}\sigma^2$$

Let,

$$(A^T A)^{-1} = \boxed{\begin{matrix} q_{xx} & q_{xy} & q_{xz} & q_{xt} \\ q_{yx} & q_{yy} & q_{yz} & q_{yt} \\ q_{zx} & q_{zy} & q_{zz} & q_{zt} \\ q_{tx} & q_{ty} & q_{tz} & q_{tt} \end{matrix}}$$

$$\text{Geometric DOP (GDOP)} = \{q_{xx} + q_{yy} + q_{zz} + q_{tt}\}^{1/2}$$
$$\text{Position DOP (PDOP)} = \{q_{xx} + q_{yy} + q_{zz}\}^{1/2}$$
$$\text{Time DOP (TDOP)} = \{q_{tt}\}^{1/2}$$

PDOP is a measure of the error due to the current spatial alignment of the satellites in view.

Other Sources of Error

	Magnitude
• Ionspheric refraction	50-150 m
• Tropospheric refraction	2-20 m
• Multipath	0.1-5m
• Relativistic	0.01-0.10m
• Other	

Differential GPS (DGPS)

- **Surveyed ground reference station**
  - **computes true range to satellites in view**
  - **computes GPS receiver measured range**
  - **calculates the differential corrections needed to adjust the receiver measured range to the actual range**

- **Broadcasts differential corrections to users**
  - **user equipment applies the corrections for much improved accuracy and precision**
  - **requires comm link; military user requires a secured link during conflict**
  - **currently used by many civil agencies including FAA and USCG**

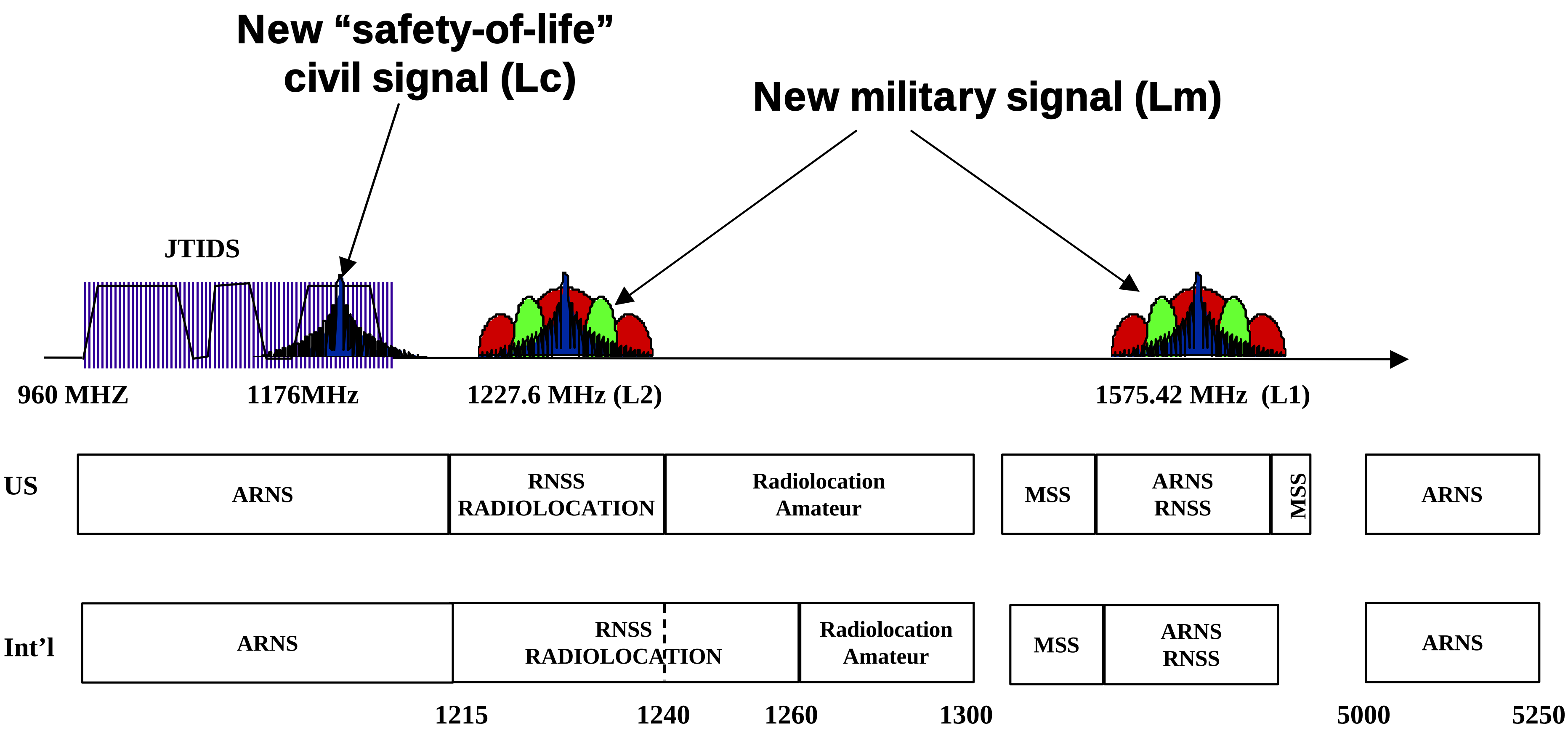
- **Civil Sector: DGPS primary method for removing SA error**

Navigational Warfare (Navwar)

- **Protection**
  - **Increase link budget**
    - **Satellite**
      - **increase transmitted power**
      - **increase transmitter antenna gain**
    - **Receiver**
      - **increase receive antenna gain**
      - **reduce sensitivity to jammers**
  - **Modify equipment for jammers**

- **Prevention**
  - **Modify selected EW platforms to meet 2006 SA turn-off date**
  - **Spectrally separate Lm from C/A signals**

New Signals


New "safety-of-life"
civil signal (Lc)
New military signal (Lm)
JTIDS
960 MHZ
1176MHz
1227.6 MHz (L2)
1575.42 MHz (L1)
US
ARNS
RNSS
RADIOLOCATION
Radiolocation
Amateur
MSS
ARNS
RNSS
MSS
ARNS
Int'l
ARNS
RNSS
RADIOLOCATION
Radiolocation
Amateur
MSS
ARNS
RNSS
ARNS
1215
1240
1260
1300
5000
5250

Selective Availability (SA)

- **Deliberately introduced reduction in accuracy**
- **Implementations**
  - **dither**
  - **epsilon**
- **A source of error for civil user**

Measurement Precision

	1 σ
SPS with SA	**100m**
SPS w/o SA	**30-50m**
PPS	**10-15m**
SPS DGPS	**5m**
PPS DGPS	**1-2m**
Carrier phase	**0.1-0.02m**

Cost Considerations

- **User equipment**
  - **stand-alone**
  - **embedded**
- **Platform integration**
- **Critical information provider to many other systems**
- **Commercial leverage**

GPS User Equipment Sales